The Binary Policy Model

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Abstract

We introduce a tractable multi-issue model of electoral competition in which candidates are exogenously committed to particular positions on a subset of issues, while they can choose a sequence of binary positions for the remaining issues to maximize their winning probability. A majority-efficient position is defined as one where a candidate cannot make a majority of the electorate better off, taking as given his fixed positions. We characterize conditions for majority-efficient positions to exist. In contrast to models where candidates can choose all relevant positions, the candidates' fixed positions in our framework imply that only some voters are *swing voters*. Whether candidates choose majority-efficient or majority-inefficient positions depends on properties of the distribution of these swing voters. We also use our framework to analyze plurality rule and runoff rule in elections with multiple candidates.

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1 Introduction

The one-dimensional policy model with office motivated candidates based on the seminal contributions of Hotelling (1929) and Downs (1957) is the most widely used and successful model framework for a formal analysis of political equilibria. We will call this model the *standard model* in the following. Yet, there are some tensions within the model, and between the model and some real-world observations.

First, in the one-dimensional standard model, there is a strong tendency for candidates to converge to the same, moderate position that appeals to the "median voter", mitigated only if the candidates care about policy and to the extent that the position of the median is uncertain. Furthermore, all voters, including those with extreme preferences, are, in equilibrium, indifferent between the two candidates, as they propose the same policy. Yet, in reality, candidates often run on considerably divergent policy platforms, and voters often intensely favor one candidate over the other. Second, while the standard model is one-dimensional and continuous, in reality, there are many policy issues, but each issue allows for only a limited number of distinct positions.

We develop a model in which policy is multidimensional and binary. Each dimension corresponds to a position on a particular policy issue, and each voter has a preferred position on each issue. A voter's utility from Candidate j is calculated by identifying those issues in which candidate and voter agree, weighing them with a factor to measure the importance of each issue, and adding up. Each candidate is exogenously fixed on some issues. Fixed positions can be interpreted as characteristics of the candidate (such as party affiliation, incumbency, gender, race or experience in previous elected office), or political issues on which a candidate has taken a stand in the past and where commitment to a different position is not credible and/or not helpful. On the remaining issues that are not fixed, candidates are free to choose any position. By developing a framework in which candidates have both fixed and selectable positions, we provide a middle ground between Downsian models, in which candidates are free to choose any position, and the citizen candidate model, in which no commitment is possible.[1] Moreover, we show that this combination enhances our understanding of competition between political candidates significantly: Some core efficiency results of existing models depend on the (seemingly innocuous and certainly unrealistic) assumption that all candidates share the same characteristics or positions on fixed issues.

The most important result for political competition in the standard model is that office-motivated candidates propose policies that appeal to the median voter. In our multi-dimensional model, there is no geometric notion of a median, but our concept of majority-efficiency captures the same fundamental idea of moderation. A policy is *majority-efficient* if a majority of voters prefers the proposed policy to any other policy that a candidate could choose. In other words, a majority-efficient policy is a Condorcet winner among a candidate's policies, subject to the constraints imposed by his fixed positions.

We show that majority-efficient policies exist in the binary policy model for many distributions of voter preferences, and we characterize necessary and sufficient conditions for existence. If a majority-

[1] In the citizen candidate literature pioneered by Osborne and Slivinski (1996) and Besley and Coate (1997), candidates are policy motivated and cannot commit to any other position than their ideal one. While the citizen candidate model can, in principle, handle multiple policy dimensions, most papers in this literature only look at a standard one-dimensional framework.

efficient position exists for a particular voter distribution, it generically also exists for a slightly perturbed distribution of voters. These existence results are interesting in their own right. More importantly, they show that our central question — namely, whether candidates choose majority-efficient policies in equilibrium — is meaningful in our framework. To include at least one fixed and at least one flexible issue dimension, our model is necessarily multidimensional. However, the multidimensional version of the standard model has the problem that a pure equilibrium generically does not exist (see Plott (1967)). In that framework, it would therefore not be surprising that candidates rarely choose majority-efficient policies. It is therefore essential for the interpretation of our results that majority-efficient policies often do exist in the binary policy model.

The key to the existence result is the discreteness of our policy space. We choose a binary setup (rather than a more general one where candidates have some finite number of feasible positions on each issue) mainly for notational convenience, and because this is the simplest multidimensional framework for the analysis of the effects of fixed positions. Moreover, while many economists are used to continuous choice variables, we would argue that a setup with very few feasible positions on each issue is actually a quite realistic description of political campaigns. A binary framework is implicitly behind several internet-based political comparison programs. For example, smartvote.ch (a cooperation project of several Swiss universities) collects the political positions of candidates in national elections by asking candidates a number of yes/no questions on different political issues. Voters can answer the same questions on a website (and also choose a weight for each issue) and are given a list of those candidates who agree with them most.[2] The restriction to a limited number of possible positions can be thought of as follows: Candidates can only communicate a limited amount of information in the campaigns and therefore can commit only to clearly defined positions. In fact, candidates are typically not very successful communicating nuanced positions to the electorate: Kerry's attempt, in 2004, to explain his preferences over different types of funding the Iraq war to voters ("I voted for the 84 Billion Dollars before I voted against them") demonstrates that point. Voters often like to know clearly where a candidate stands on the issues. An alternative interpretation, which is backed up by experiments in psychology, is that voters organize information in broad categories and have a limited ability to understand and remember differences in policy proposals (see, for example, Mullainathan, Schwartzstein, and Shleifer (2008) and the references therein). In this case, candidates can only choose to position themselves in one of several categories.

In the standard model, the equilibrium policy is obviously majority-efficient. The same is true in the binary policy model if there is no difference between the candidates' fixed positions. In contrast, however, the multidimensional nature of the binary policy model fundamentally changes this, if candidates have different fixed positions. In equilibrium, a candidate may propose majority-inefficient policies, because adopting minority positions may *increase* his winning probability.

If candidates differ in their fixed positions, some voters will strictly prefer one of the candidates, no matter which policies the candidates choose on flexible issues. The candidates effectively only compete for the votes of the remaining *swing voters* (i.e., those individuals whose vote depends on the policies

[2]Similar programs exist for Germany (http://www.wahl-o-mat.de), Austria (http://www.wahlkabine.at/) and the Netherlands (http://www.stemwijzer.nl/)

proposed by the two candidates). We identify two fundamental reasons for inefficiency results. First, the preference distribution among swing voters may differ from that of the population at-large. Since candidates care about pleasing swing voters, they may do so even if such a position goes against the wishes of the majority of the overall electorate.

Second, and more surprisingly, majority-inefficient choices can even arise if the preference distribution among swing voters is the same as in the population at large. The reason is that candidates compete for different groups of swing voters. If Candidate 0 has fewer swing voters who prefer him than Candidate 1, if both choose the same policy on flexible issues, then Candidate 0 may benefit if he chooses to deviate to a minority position. While he will lose a majority of his previous swing voter support, and win only a minority of his opponent's previous supporters, the asymmetry between different swing voter groups implies that the effect on Candidate 0's total vote, and thus on his probability of winning, may still be positive. We also show that asymmetric swing voter distributions arise very naturally even in a setting where all voters place the same weight factors on the different issues, and ideal positions are independently distributed.

Our model also contributes to the interpretation of policy divergence. In the standard model, neglecting the median voter in order to choose policies that please minorities reduces a candidate's winning probability, so that platform divergence cannot be rationalized as an electoral success strategy in that model. The observation that political candidates often propose divergent policies has been interpreted in the sense that the fundamental assumptions in the basic standard model, in particular policy motivation of candidates, have to be modified in order to be consistent with observed candidate behavior. For example, policy motivation of candidates and uncertainty about the position of the median voter can generate policy divergence. In contrast, in the binary policy model, policy divergence can arise in a complete information setting with purely office-motivated candidates.

The binary policy model also yields novel results when comparing the performance of plurality rule and runoff rule electoral systems in elections with more than two candidates. In most models in the literature, candidates can either commit on all issues, or on none; in this case, we show that runoff rule weakly dominates plurality rule: There are voter preference distributions such that a majority of voters prefers the election outcome under runoff rule to the election outcome under plurality rule, but never the reverse. In contrast, if candidates differ in their fixed positions and can choose positions on some other issues, then the set of swing voters is smaller than the total electorate. We show that there are instances in which candidates choose more moderate positions under plurality rule, and where a majority of voters strictly prefers the election outcome under plurality rule to the one under runoff rule. This result applies more generally. While one electoral system may be more desirable than some other electoral system *for any given set of candidates* (in the sense of selecting more often a candidate who is preferred by a majority of the electorate), this relation may reverse once we take into account the effect of the electoral system on the positions that candidates take. Thus, focusing on the likelihood of selecting the Condorcet winner among a given set of candidates in comparing the performance electoral systems is inherently problematic.

We present the model in the next section. In Section 3, we introduce the concept of majority-efficiency and derive the main results for competition between two candidates. Section 4 analyzes plurality rule and runoff rule in the binary policy model with three candidates. Section 5 concludes. All proofs are in the Appendix.

2 The model

2.1 Setup

Two candidates, $j = 0, 1$, compete in an election. Candidates are office-motivated and receive utility 1, if elected, and utility 0, otherwise, independent of the implemented policy. There are I issues, and the set of issues is denoted by $\Im = \{1, \ldots, I\}$. Candidate j, if elected, implements a policy described by $a^j = (a_i^j)_{i\in\Im} \in A = \{0, 1\}^I$, where each $a_i^j \in \{0, 1\}$ denotes Candidate j's position on issue i (0 can be interpreted as opposition to a particular proposal, and 1 as support of that proposal).

Candidate j can freely choose a policy on a subset of issues $S^j \subset \Im$, while no commitment is possible on the remaining issues. Thus, *Candidate j's type* is given by $(a_i^j)_{i\notin S^j}$, while his *platform* is $(a_i^j)_{i\in S^j}$. Candidate j's *policy* consists of the combination of his type and platform, so that his set of feasible policies is given by $A^j = \{(a_i)_{i\in\Im} | a_i = a_i^j$ for all $i \notin S^j$ and $a_i \in \{0, 1\}$ for $i \in S^j\}$.

Let T be the set of voter preference types, with typical element $\tau = (\theta, \lambda) \in A \times \mathbb{R}_+^I$. We allow T to be finite or infinite. Each voter type τ has preferences on A of the form

$$u_\tau(a) = -\sum_{i=1}^{I} \lambda_i |\theta_i - a_i|. \tag{1}$$

We refer to such preferences as "weighted issue preferences." The citizen has an ideal position a_i on each issue i and the importance of issue i is given by the weight λ_i.

Let μ be the distribution of voter types. Note that this is just a frequency distribution that is known to the candidates. If T is finite then $\mu(\{\tau\})$ is the percent of voters in the population that are of type τ. The timing of the game is as follows:

Stage 1 Candidates $j = 1, 2$ simultaneously announce policies $a^j \in A^j$. A mixed strategy by agent j consists of a probability distribution σ^j over A^j.

Stage 2 Each individual votes for his preferred candidate, or abstains when he is indifferent between both candidates.[3] Candidate j wins if $\mu\left(\left\{\tau | a^j \succ_\tau a^{-j}\right\}\right) > \mu\left(\left\{\tau | a^j \prec_\tau a^{-j}\right\}\right)$. In case of a tie between the candidates, each wins with probability 0.5.

Clearly, mixed strategy equilibria always exist since each S^j is finite.

2.2 Interpretation

We assume that, in each issue, candidates have (or take) one of two positions. A binary model is the simplest way to capture the more general idea that the policy space consists of a set of finite categories from

[3]If a voter has a strict preference, then it is a weakly dominant strategy to vote for the preferred candidate. If an agent is indifferent, he could in principle vote for any candidate or abstain, but the assumption of abstention is quite natural, and none of the results in this paper depends critically on it.

which policy can be chosen, and another set on which candidates' positions are already determined. For issues where the candidates' positions are fixed, the binary setup is without loss of generality even if the space of generally feasible positions or characteristics is larger. Consider, for example, voters' ethnic or racial preferences. Each candidate belongs to one of several different races, and individual voters have a (possibly strict) preference ranking over all realizations of this characteristic. However, if (say) one of the candidates is white, while the other one is African-American, then it is irrelevant how voters would feel about, say, an Asian candidate. Thus, for a given pair of candidates, we can model the racial characteristics of candidates as binary. This argument applies more generally for fixed positions.

The utility function (1) assumes that the issues enter in voters' utility functions in a separable way, so that the position that is adopted on issue i does not affect a voter's preferred policy position on issue j. This assumption appears reasonable if we consider two completely separate policy issues. For example, if a voter prefers school vouchers to be provided if candidates oppose gun control, then this preference should not change if the candidates support gun control.[4] However, there are also cases in which complementarities between issues can yield non-separable utility functions. For example, in Krasa and Polborn (2009), we analyze a model in which a voter's preferred policy from a candidate depends on the candidate's level of expertise, which is a fixed characteristic. In summary, we think of weighted-issue preferences in the present model as a useful and simple benchmark, rather than as a necessarily realistic assumption in all circumstances.

A key ingredient of our model is the mix between fixed positions (as in the citizen-candidate model) and flexible position as in the original Downsian model with office motivation. The notion of fixed positions in our model is related to the valence literature that analyzes the effect of a non-policy candidate characteristic such as competence that cannot be chosen by the candidates. Indeed, Groseclose (2001), p. 862, argues that "if either candidate has an entrenched position on a past policy issue, this may work like a valence advantage". However, valence is a very special fixed characteristic in that it is appreciated by all voters, while a particular fixed position on a policy issue generally makes a candidate more popular with some voters and less popular with others. Our results will show that this difference matters. In particular, being fixed to the majority-preferred position while the opponent is fixed to the minority position is *not* equivalent to having some valence advantage.

3 Majority-Efficiency

3.1 Definition

The central result for political competition in the standard framework is that candidates propose policies that appeal to the median voter. This median voter result corresponds to a notion that political competition forces candidates to propose "popular" policies in order to win elections. "Moving toward the median" (from a non-median initial position) is popular in the standard model because it is preferred by

[4]Note that we do not assume anything about the correlation of ideal position on different issues. For example, it could be the case that voters who support school vouchers are more likely to oppose gun control than those who oppose school vouchers.

a majority of voters. While there is no geometric notion of a "median voter" in our model, the following concept captures the same fundamental idea. A candidate's policy is *majority-efficient* if a majority of voters prefers this policy to any other policy that the candidate could choose. Thus, a majority-efficient policy is the most popular policy a candidate can choose, subject to the constraints imposed by his fixed positions. The central question of our analysis is whether candidates choose majority-efficient positions in equilibrium. We first define majority preferences over policies.

Definition 1 *Policy a is **majority-preferred** to a', denoted by $a \succeq^* a'$, if and only if $\mu(\{\tau | a \succeq_\tau a'\}) \geq \mu(\{\tau | a' \succeq_\tau a\})$.*

*Furthermore, if $a \succeq^* a'$ but not $a' \succeq^* a$, we say that a is **strictly majority-preferred** to a', denoted by $a \succ^* a'$.*

A policy is majority-efficient if and only if it is a Condorcet winner *relative to the set of the candidate's feasible policies*.

Definition 2 *Candidate j's policy $a^* \in A^j$ is **majority-efficient** if and only if $a^* \succeq^* a$ for all $a \in A^j$.*

In general, the smaller is the set of feasible policies, the more likely it is that a majority-efficient policy in such a set exists. In fact, it is straightforward to find examples in which majority-efficient policies exist, but there is no Condorcet winner in the traditional sense (where all positions can be chosen). Thus, recognizing that candidates are in practice fixed on many positions, and restricting the comparison set to a candidate's feasible policies, makes majority-efficiency a more applicable concept than that of a Condorcet winner.

The reader may wonder whether majority-efficiency as a candidate-specific concept is necessarily related to welfare properties of the equilibrium. For example, if a candidate is never elected in any equilibrium, then his position is irrelevant and so it does not matter whether his proposed policy is majority-efficient. However, it is easy to see that if a candidate is a serious contender, then majority-efficiency is equivalent with the winning policy being a Condorcet winner among all feasible policies of all candidates. That is, in a pure strategy equilibrium, there is no feasible policy for either candidate that would make a majority of the voters better off if and only if the winner's platform is majority-efficient. More formally we state this insight in the remark below.

Remark 1 *Let (a^0, a^1) be a pure strategy equilibrium and assume without loss of generality that Candidate 0 wins the election, so that policy a^0 is implemented. There is no $\tilde{a} \in A^0 \cup A^1$ such that $\tilde{a} \succ^* a^0$ if and only if a^0 is majority-efficient.*

Note that we do not claim that the candidates' equilibrium strategies are necessarily majority-efficient. Indeed, we will show that this is not always the case, even in pure strategy equilibria. Thus, in these cases the outcome is undesirable from a social perspective.

3.2 Existence

We now show that majority-efficient policies exist in all propositions and examples considered in this paper. These existence results are interesting in their own right, but also are essential for the interpretation of our main results in Sections 3.3 and 4, which deal with the question whether candidates adopt majority-efficient policies in equilibrium. By showing that existence of majority-efficient positions is robust in the binary policy model, we can conclude that adoption of majority-inefficient positions is not caused by lack of existence of a majority-efficient policy.

Lemma 1 relates majority-efficiency to a property of the median of the distribution of ideal points, weighted by issues weights λ_i.

Lemma 1 *Suppose that preferences are given by (1) and that all voters have the same issue weights λ_i for selectable issues $i \in S^j$. Let $\overline{a}^j \in A^j$ and for every $i \in S^j$ define $X_i(\theta) = 1$ if $\theta_i = \overline{a}_i^j$ and $X_i(\theta) = 0$, otherwise. Then $\overline{a}^j$ is majority-efficient if and only if* $\text{median}\left(\sum_{i \in D} \lambda_i X_i\right) \geq 0.5 \sum_{i \in D} \lambda_i$ *for all $D \subset S^j$.*

Intuitively, if the median of $\sum_{i \in D} \lambda_i X_i$ is greater than 0.5, then at least half of the population prefers $\overline{a}$ to another policy that differs from $\overline{a}$ on issues in D. We use Lemma 1 to prove Proposition 1 below, using the fact that a distribution F that first-order stochastically dominates a distribution G has a higher median.

First, however, we apply Lemma 1 to the case of only two selectable issues. Under which conditions is choosing 1 on both issues majority-efficient? By Lemma 1, the median of the weighted distribution of ideal points, $\sum_{i \in D} \lambda_i \theta_i$ must have a median of at least $0.5 \sum_{i \in D} \lambda_i$ for any set of selectable issues $D \subset \{1, 2\}$. If D consists of a single issue, then this property is satisfied if and only if $\mu(\{\theta_i = 1\}) \geq 0.5$, i.e., if at least 50% of the population prefer $\theta_i = 1$ to $\theta_1 = 0$. Now let $D = \{i_1, i_2\}$, and suppose (without loss of generality) that $\lambda_1 \geq \lambda_2$. Then $\lambda_1 \theta_1 + \lambda_2 \theta_2$ has four possible realizations $0 < \lambda_2 \leq \lambda_1 < \lambda_1 + \lambda_2$. The median condition is therefore satisfied as long as $\mu(\{\theta_1 = \theta_2 = 1\} \cup \{\theta_1 = 1, \theta_2 = 0\}) \geq 0.5$, which is equivalent to $\mu(\{\theta_1 = 1\}) \geq 0.5$ (and thus does not impose an additional restriction). The argument generalizes immediately for arbitrary $\overline{a}^j \in A^j$: The policy preferred by a majority on both issues is majority-efficient.

Corollary 1 *Suppose that there are two selectable issues and that all voters have the same issue weights λ_i for them. Then $\overline{a}^j \in A^j$ is majority-efficient if and only if $\mu(\{\theta_i = \overline{a}_i^j\}) \geq 0.5$.*

The significance of Corollary 1 is that a number of interesting examples can already be generated with one or two flexible issues, and in all these applications, existence is guaranteed.[5] If there is only one selectable issue then a majority-efficient policy obviously always exists (even if agents also differ by issue weights).

We now provide another result that can be used to characterize majority-efficient policies.

[5] A result similar to Corollary 1, but based on very different assumptions, is derived by Bade (2006). She shows, in a two-dimensional Euclidean model, that an equilibrium of the game between candidates exists (located at the median in each dimension), if candidates are uncertain about the shape of voters' indifference curves and are uncertainty-averse (rather than expected-utility maximizers).

Proposition 1 *Suppose that all voters have the same issue weights* λ_i, $i \in \Im$. *Let* $\overline{a}^j \in A^j$. *For every* $i \in S^j$ *define* $X_i(\theta) = 1$ *if* $\theta_i = \overline{a}_i^j$ *and* $X_i(\theta) = 0$, *otherwise, and let* F *be the distribution of* $X = (X_i)_{i \in S^j}$.

1. *If*
$$F(x) \leq \prod_{i \in S^j} 0.5^{1-x_i}, \text{ for all } x \in \{0,1\}^{S^j}, \tag{2}$$
then $\overline{a}^j$ *is majority-efficient.*

2. *Suppose that* $\mu\left(\{\theta_i = \overline{a}_i^j\} \mid \{\theta_{i_1} \neq \overline{a}_{i_1}^j, \ldots \theta_{i_k} \neq \overline{a}_{i_k}^j\}\right) \geq \mu\left(\{\theta_i = \overline{a}_i^j\}\right)$, *for all* $\{i_1, \ldots, i_k\} \subset I \setminus \{i\}$. *Then the reverse implication is also true, i.e., if* $\overline{a}^j$ *is majority-efficient then* (2) *must hold.*

To get an intuition for Proposition 1, consider the case where $\overline{a}_i^j = 1$ for all flexible issues $i \in S^j$ is majority efficient. If the distribution of types θ is uniform in $\{0,1\}^I$, then $\overline{a}^j$ (as well as any other policy) is clearly majority-efficient. Now consider a new distribution of types that first-order stochastically dominates the uniform distribution (i.e., we take some voters and switch their preference on at least some issues from 0 to 1). The median of $\sum_{i \in D} \lambda_i X_i$ must now be (at least weakly) higher than under the uniform distribution (for every D), and thus (by Lemma 1), $\overline{a}$ remains majority-efficient. The second part of Proposition 1 shows that, if there is a negative correlation between the individual issues, then the condition is both necessary and sufficient.

An immediate consequence of Proposition 1 is that if the X_i are independent, that $\overline{a}^j$ is majority-efficient if and only if $\mu(\{\theta_i = \overline{a}_i^j\} \geq 0.5$ for all $i \in S$. We now state this result.

Corollary 2 *Suppose that voters have the same issue weights and that all marginal distributions of ideal points are independent, i.e.,* $\mu(\{\theta_i = a_i, \theta_j = a_j\}) = \mu(\{\theta_i = a_i\})\mu(\{\theta_j = a_j\})$ *for all* $a_i, a_j \in \{0,1\}$, $i \neq j$, $i, j \in S^j$. *Then policy* $\overline{a}^j$ *is majority-efficient if and only if* $\mu(\{\theta_i = \overline{a}_i^j\}) \geq 0.5$.

In summary, the results in this section suggest that the existence of majority-efficient policies is relatively robust in the binary policy model, in the sense that the distribution of voters can be changed in a generic way without affecting the existence of a majority-efficient policy. This robustness result contrasts with the generic non-existence of equilibrium multidimensional Euclidean model of Plott (1967).[6]

3.3 Adoption of minority positions in the binary policy model

Identical fixed positions. To our knowledge, all existing deterministic voting models in which a majority-efficient platform exists have the feature that both candidates choose that policy as their equilibrium platform. This is certainly true for the one-dimensional Downsian model, in which both candidates choose the median voter's bliss point. Also, while a majority-efficient position rarely exists in

[6]In Plott (1967), each voter is indifferent between all policies that have the same distance from his bliss point, and a majority-efficient policy corresponds to a Condorcet winner (as there are no fixed positions). Plott shows that a Condorcet winner exists if and only if the distribution of voter ideal points is radially symmetric around one voter's ideal point (i.e., that voter is the "median in all directions"). This existence condition is highly non-generic: Starting from a radially symmetric distribution and changing the ideal point of only one voter usually destroys radial symmetry. This is true even if, in the spirit of our Proposition 1, we move that voter's ideal point closer to the previous median, as long as we don't move it exactly on the line that connects the median with the voter's previous ideal point.

a multidimensional Downsian model, if it does, then both candidates choose it as a platform. These results suggest that, if candidates can choose majority-efficient positions, they will always do so in equilibrium. Perhaps surprisingly, this conjecture turns out to be false in the binary policy model, if candidates differ in their fixed positions. Only if candidates have exactly the same fixed positions, then they are guaranteed to choose a majority-efficient platform, provided that one exists.

Proposition 2 *Suppose that $A^0 = A^1$:*

1. *Then (a^0, a^1) is a pure strategy Nash equilibrium if and only if both a^0 and a^1 are majority-efficient.*

2. *Suppose a majority-efficient policy exists. If (σ^0, σ^1) is a mixed strategy equilibrium, then every policy in the support of σ^0 and σ^1 is majority-efficient.*

Proposition 2 highlights the role of fixed positions for all majority-inefficiency results in this paper, and indeed, for related results when voters do not necessarily have weighted-issue preferences.[7] One interesting issue is, for example, whether there are any models in which office-motivated candidates have a strict incentive to differentiate from their opponent for electoral gain.[8] Proposition 2, point 1, shows the crucial role of fixed positions for any such model. If there are no differences between the candidates' fixed positions, then a pure strategy equilibrium with differentiation exists only if there are two (or more) majority-efficient policies. Moreover, even in this case (say, there are two majority-efficient policies, a and b), there are also equilibria in which both candidates choose the same policy (that is, (a, a) and (b, b) are equilibria), and in all equilibria, candidates are indifferent between playing a and b, so there is never a strict incentive for candidates to differentiate. The same is obviously true for symmetric mixed strategy equilibria.

Swing voters. We now analyze whether candidates select majority-efficient platforms if they differ in their fixed positions. Our main points can already be made in a very simple setup where both candidates are flexible on only one (and the same) issue. However, it will be clear that the basic principles identified here do not relie on this simple structure, but apply more generally.

Without loss of generality, assume that both candidates are flexible on issue I, while they are fixed in all of the first $I-1$ issues. It is useful to define the notion of a swing voter as a marginal supporter. We say that a voter is a *swing voter for Candidate* 1 if he prefers Candidate 1 if both candidates choose the same policy on issue I, but prefers Candidate 2 if Candidate 2 proposes the voter's preferred position on issue I while Candidate 1 proposes the opposite position.

Definition 3 *Voter type $\tau = (\lambda, \theta)$ is a swing voter for Candidate j if*

$$-\sum_{k=1}^{I-1} \lambda_k |\theta_k - a_k^{-j}| < -\sum_{k=1}^{I-1} \lambda_k |\theta_k - a_k^{j}| < -\sum_{k=1}^{I-1} \lambda_k |\theta_k - a_k^{-j}| + \lambda_I. \tag{3}$$

[7] The proof of Proposition 2 applies for general voter preferences, not just for weighted-issue preferences.

[8] We are grateful to Ernesto Dal Bo for raising this question.

Let SV_j *denote the number of swing voters for Candidate* j*, i.e,* $\mathrm{SV}_j = \mu\Big(\{\tau \,\Big|\, \tau \text{ satisfies } (3)\}\Big)$*. Furthermore, let* $\xi_j = \mu\Big(\{\tau|\tau \text{ satisfies } (3) \text{ and } \theta_I = 0\}\Big)$ *denote the percentage of voters who prefer position 0 in issue I among the swing voters of Candidate* j*.*

A voter is either a swing voter, or a *core supporter* of one of the candidates (i.e., prefers the fixed positions of one candidate so much that he would never vote for his opponent, independent of the candidates' positions on the flexible issue). Note that, without fixed issues, *all* voters are swing voters.

Obviously, equilibria with minority positions can occur if one candidate is so much stronger than his opponent that he wins, no matter what his policy positions are. To rule out such trivial cases of majority-inefficient equilibria, we introduce the following refinement:

Definition 4 *An equilibrium* (a^0, a^1) *satisfies the* vote-maximization property *if there is no strategy for either candidate that increases the number of votes he receives.*

A way of justifying this refinement is to embed our model in one where there is, in addition to the rational voters modeled here, a random number of "noise voters" who vote for Candidate 0 with probability p and for Candidate 1 with probability $1 - p$, irrespective of the policy positions that the candidates take. In such a framework, both candidates maximize their overall winning probability by maximizing the number of votes they receive from rational voters.

Since the number of core-supporters does not depend on the platforms, we can focus on swing voters for the determination of vote-maximizing equilibria. The number of swing voters who vote for the two candidates, depending on their platforms, is given by Figure 1. For example, if Candidate 0 plays 0 and Candidate 1 plays 1, then all swing voters who prefer 0 on issue I vote for Candidate 0 and vice versa.

		Cand. 1	
		0	1
Cand. 0	0	$\mathrm{SV}_0, \mathrm{SV}_1$	$\xi_0\mathrm{SV}_0 + \xi_1\mathrm{SV}_1, (1-\xi_0)\mathrm{SV}_0 + (1-\xi_1)\mathrm{SV}_1$
	1	$(1-\xi_0)\mathrm{SV}_0 + (1-\xi_1)\mathrm{SV}_1, \xi_0\mathrm{SV}_0 + \xi_1\mathrm{SV}_1$	$\mathrm{SV}_0, \mathrm{SV}_1$

Figure 1: Swing voters voting for Candidate 0 and 1

The following Proposition 3 states necessary and sufficient conditions for equilibria that satisfy the vote-maximizing property.

Proposition 3 *Suppose that candidates are flexible only in issue I.* $(0, 0)$ *is an equilibrium that satisfies the vote-maximizing property if and only if*

$$\frac{\mathrm{SV}_1}{\mathrm{SV}_0} \in \left[\frac{1-\xi_0}{\xi_1}, \frac{\xi_0}{1-\xi_1}\right]. \tag{4}$$

$(1, 1)$ *is an equilibrium that satisfies the vote-maximizing property if and only if*

$$\frac{\mathrm{SV}_1}{\mathrm{SV}_0} \in \left[\frac{\xi_0}{1-\xi_1}, \frac{1-\xi_0}{\xi_1}\right]. \tag{5}$$

$(0,1)$ *and* $(1,0)$ *are equilibria that satisfy the vote-maximizing property if and only if*

$$\mathrm{SV}_1 = \mathrm{SV}_0 \textit{ and } \xi_0 + \xi_1 = 1. \tag{6}$$

In the latter case, $(0,0)$ *and* $(1,1)$ *are also equilibria.*

Clearly, (6) holds only in highly non-generic circumstances. Disregarding this case, at most one of equations (4) and (5) can hold, since the lower limit of the interval in (4) is the upper limit of the interval in (5), and vice versa (so that, generically, one of the two intervals is an empty set, as its lower limit is a higher number than its upper limit). Without loss of generality, suppose that the interval on the right hand side of (4) is non-empty. The proposition tells us that, for $(0,0)$ to be an equilibrium, the ratio of swing voters must neither be too low nor too high. We now use Proposition 3 to identify two fundamentally different incentives for candidates to choose majority-inefficient policies in equilibrium.

Non-representative swing voters. The first reason for equilibrium majority-inefficiency is quite straightforward: The preferences of swing voters (as captured by ξ_0 and ξ_1) may not be representative for the preferences of the population at large. The following example illustrates this point.

Example 1 There are two issues. In the first issue, Candidate 0 is fixed at 0 and Candidate 1 at 1. Both are flexible on issue 2. Voter types are given by $(\lambda_1, \lambda_2, \theta_1, \theta_2)$. Suppose there are only four types $T = \{\tau_1, \tau_2, \tau_3, \tau_4\}$, where $\tau_1 = (\overline{\lambda}, \underline{\lambda}, 0, 0)$, $\tau_2 = (\overline{\lambda}, \underline{\lambda}, 1, 0)$, $\tau_3 = (\underline{\lambda}, \overline{\lambda}, 0, 1)$, and $\tau_4 = (\underline{\lambda}, \overline{\lambda}, 1, 1)$, and $\overline{\lambda} > \underline{\lambda}$. That is, τ_1 and τ_2 consider the first, fixed issue, to be the more important one and therefore are core supporters of the candidates. In particular, τ_1 always votes for Candidate 0 and τ_2 always votes for Candidate 1, no matter what policies the candidates choose on issue 2. In contrast, τ_3 and τ_4 put a high weight on issue 2 and are thus the swing voters for Candidate 0 and 1, respectively: If candidates choose different positions on issue 2, they would vote for the candidate who picks their preferred position.

Since all swing voters prefer $a_2 = 1$ (i.e. $\xi_0 = \xi_1 = 0$), it is clear that both candidates choose $a_2 = 1$ in the unique vote-maximizing equilibrium. Candidate 0 receives the votes of types τ_1 and τ_3, while Candidate 1 receives the votes of τ_2 and τ_4. This equilibrium is not majority-efficient, if a majority of voters prefers position 0 on the second issue (i.e., if $\mu(\{\tau_1\} \cup \{\tau_2\}) > \mu(\{\tau_3\} \cup \{\tau_4\})$. ■

The effect of Example 1 is also present in the probabilistic voting model (henceforth PVM) pioneered by Lindbeck and Weibull (1987),[9] in which both candidates choose the same policy platform that maximizes a weighted sum of utility of different groups in society. The weight of a group in the candidates' objective function is higher than its population share, if voters in the group are more "movable", i.e. if they are relatively likely to switch to a candidate who offers them a more favorable policy position. Example 1 is based on a analogous effect, but provides the result in a simpler and deterministic setting: In the PVM, there are a number of exogenous random shocks, and members of a group with the

[9]In the PVM, voters are divided in different groups according to their utility from policies (that are choice variables for candidates). In addition, all voters receive a common utility shock (like valence), and an idiosyncratic "ideology" shock. See also Lindbeck and Weibull (1993), Coughlin (1992) or Persson and Tabellini (2000) for a review of the various developments of this literature.

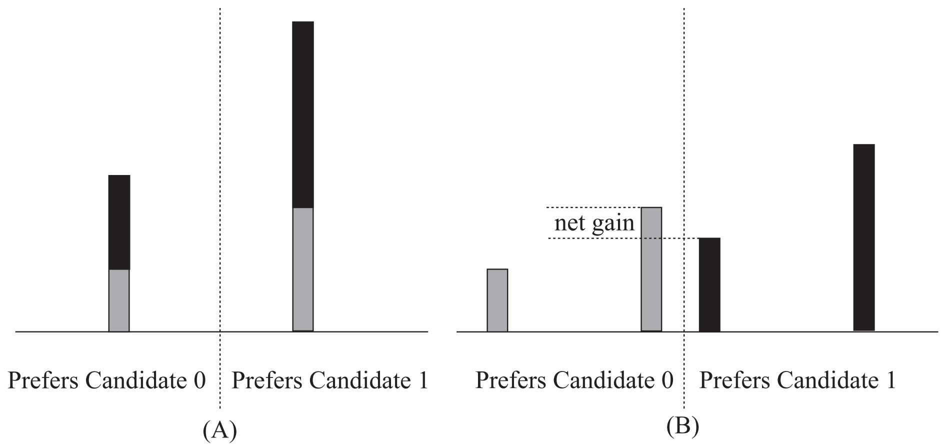

Figure 2: Net Preference Distribution for Candidate 1

same interests about issues must be sufficiently differentiated "ideologically" (i.e., in a dimension that cannot be addressed by the candidates) for an equilibrium to exist.[10]

Asymmetric swing voter distribution. Even if preferences of swing voters are representative, majority-inefficient policies may be adopted. For example, suppose that 60% of the population prefers issue 0, and that the same is true among swing voters, i.e., $\xi_0 = \xi_1 = 0.6$. If the distribution of swing voters is sufficiently balanced (i.e., if $2/3 \leq \mathrm{SV}_1/\mathrm{SV}_0 \leq 1.5$), then, $(0, 0)$ is an equilibrium. However, if Candidate j has 1/3 fewer swing voters to defend than his opponent, then Candidate j will deviate and select position 1 on issue I.

Consider Figure 2, where we focus only on swing voters. Panel (A) shows the distribution of swing voters when both candidates adopt the majority-efficient position 0 on issue I. The black part of each bar indicates the voters who prefer 0 on issue I, whereas the gray portion stands for the minority that prefers position 1. All swing voters to the left of the dashed vertical line vote for Candidate 0, while those to the right vote for Candidate 1. Now suppose that Candidate 0 adopts policy 1 on issue I. Then he loses the black parts of both swing voter groups, and wins both gray parts. Since Candidate 1 starts out with more swing voters in (A), this results in the net gain of votes for Candidate 0 indicated in (B). Moreover, it is clear that this net gain of votes may also be enough to swing the election, in which case the deviation to majority-inefficient policies strictly increases the payoff of Candidate 0.

To analyze this effect in more detail, consider a situation in which the distribution of preferences over issues is independent (as in Corollary 2). Thus, the percentage of voters in the general electorate who prefer position 0 on issue I is equal to the corresponding percentage among both swing voter groups, i.e. $\mu(\{\tau|\theta_i = 0\} = \xi_0 = \xi_1$. Proposition 4 first provides sufficient conditions that guarantee that

[10]Besley and Coate (2001) present a model in which parties choose candidates to maximize the utility of the majority of their members. The electorate consists of rational voters (who are influenced by the candidates' platforms) and noise voters who vote randomly. Besley and Coate show that parties may choose majority-inefficient positions in equilibrium if there are no swing voters, or if a majority of rational swing voters prefer the minority position.

a candidate can win by choosing a majority-efficient policy. In general, each candidate will have some "strong" characteristics, i.e., those issues on which a majority prefers his fixed positions. Intuitively, a candidate is the better, the more important these issues are and the higher the majority that supports the candidate's position. Suppose that issues can be paired such that, for each issue in which Candidate 0 is weak, there is another, more important (in terms of λ) issue in which he is strong, and the majority preferring Candidate 0 in his strong issue is larger than the majority favoring his opponent in Candidate 0's weak issue. In this case, Proposition 4 shows that Candidate 0 can win by choosing a majority-efficient policy.

In contrast, if such a ranking of candidates is not possible, the distribution of swing voters can be sufficiently asymmetric such that the candidate who would lose the election if both candidates adopt the majority-efficient position can win by choosing a majority-inefficient policy. Obviously, both candidates choosing a majority-efficient policy is then not an equilibrium. Moreover, the cases are robust in the sense that, for any distribution of ideal points, there exists an open set of issue weights for which this phenomenon arises.

Proposition 4 *Suppose that voters have the same issue weights so that type space $T = \Theta$. Assume that all marginal distributions μ_{Θ_i} are independent, and that $S^0 = S^1 = S$. Suppose that, for each issue $i \in \mathcal{I} \setminus \mathcal{S}$ on which Candidate 1 is fixed to the majority preferred position (i.e., $\mu(\{\theta|\theta_i = a_i^1\}) \geq 0.5$), there exists an issue $\phi(i) \in \mathcal{I} \setminus \mathcal{S}$ on which Candidate 0's position is preferred by a larger majority (i.e. $\mu(\{\theta|\theta_{\phi(i)} = a^0_{\phi(i)}\}) > \mu(\{\theta|\theta_i = a_i^1\})$), where ϕ is a one-to-one mapping.*

1. *If $\lambda_{\phi(i)} > \lambda_i$ for all i with $\mu(\{\theta|\theta_i = a_i^1\}) \geq 0.5$ then Candidate 0 wins by choosing a majority-efficient policy.*

2. *Let $j \in S$. There exists an open set $\Lambda \times M_j \subset \mathbb{R}_+^I \times (0.5, 1]$ such that for all utility weights $(\lambda_1, \dots, \lambda_I) \in \Lambda$ and all μ with $\mu(\{\theta|\theta_j = a_j\}) \in M_j$, the following holds:*
   If Candidate 1 selects a majority-efficient policy, then Candidate 0 wins if and only if he selects a majority-inefficient policy.

To get an intuition for Proposition 4, it is useful to consider special cases with only a few fixed positions. First, consider the case of only one fixed position. (In order to make this situation fit the condition in Proposition 4, we can add a spurious second issue with $\lambda_2 = 0$). It is easy to see that Candidate 0, who has the advantage on the fixed issue, can guarantee himself a victory by choosing the majority-preferred position on the flexible issue: His opponent can either also choose the majority-preferred position on issue 3, in which case Candidate 0 wins with the support of the majority that prefers his fixed position; or Candidate 1 can take the other position on issue 3, in which case, either nothing changes (if $\lambda_1 > \lambda_3$), or, if $\lambda_1 < \lambda_3$, Candidate 0 is supported by the majority of people who prefer the majority-preferred position on issue 3.[11]

Second, consider the case of two fixed positions and one flexible one. Suppose that Candidate 0 is fixed to $(0, 0)$, while Candidate 1 is fixed to $(1, 1)$. Except for relabeling of candidates, there are

[11]A special case of this example is a model with valence differences between candidates (i.e., *all* voters prefer the "fixed position of one candidate over the one of his competitor).

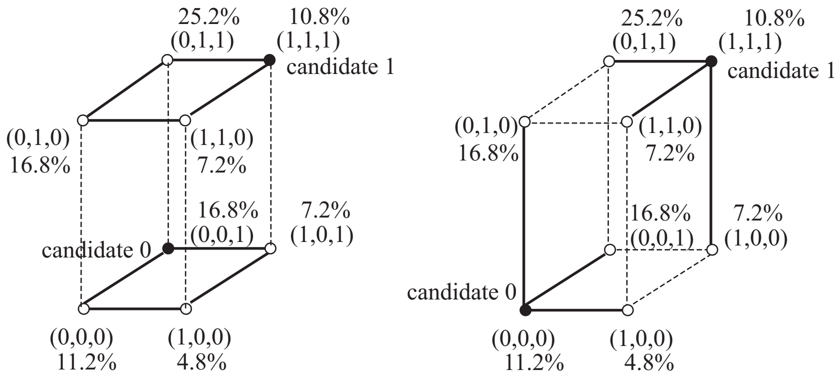

Figure 3: Illustration of Example 2

three possibilities: (a) for both issues 0 is preferred by majority of voters; (b) a majority prefers 0 on the first and a smaller majority prefers 1 on the second issue, and $\lambda_1 > \lambda_2$; (c) a majority prefers 0 on the first and a smaller majority prefers 1 on the second issue, and $\lambda_1 < \lambda_2$. Proposition 4 implies that Candidate 0 wins using a majority-efficient position on the third issue in cases (a) and (b), no matter what Candidate 1 does. We now focus on case (c) which provides an example for the second statement of Proposition 4.

Example 2 Suppose that $\mu(\tau|\theta_1 = 0) = 0.7$ and $\mu(\tau|\theta_2 = 1) = \mu(\tau|\theta_3 = 1) = 0.6$, that $\lambda_1 < \lambda_2$ and that $\lambda_2 - \lambda_1 < \lambda_3 < \lambda_2 + \lambda_1$. Thus, issue 3 will not sway voters who prefer one of the candidates in both of the first two issues, but has potentially an effect on those who have mixed preferences on the first two issues. The percentages of voters of each type are given in Figure 3. Candidate 0 is fixed to 0 and Candidate 1 to 1 on the first two issues, i.e., Candidate 1 can choose between $(0, 0, 0)$ and $(0, 0, 1)$, while Candidate 1 can choose between $(1, 1, 0)$ or $(1, 1, 1)$. In Figure 3, a citizen votes for the candidate whose policy is closer in distance, where the distance in the vertical (second) dimension is larger than those in the other two directions.

Note that 60% prefer Candidate 1's position on issue 2 which is more important to voters because of the higher weight, but an even larger majority of 70% prefers Candidate 0 on the less important issue 1. If both candidates adopt the majority-efficient position $a_3 = 1$, then Candidate 1 receives the votes of all $\theta_2 = 1$ types, because issue 2 is more important to voters than issue 1, and wins with 60% of the vote (see left panel of Figure 3). Types $(1, 0, 0)$ and $(1, 0, 1)$ prefer Candidate 0 to Candidate 1 even though they are the same number of nodes away from either candidate, because the vertical distance is larger. For similar reasons, types $(0, 1, 0)$ and $(0, 1, 1)$ are better off with Candidate 1.

Note that these four voter types are the swing voters for Candidate 0 ($(1, 0, 0)$ and $(1, 0, 1)$) and Candidate 1 ($(0, 1, 0)$ and $(0, 1, 1)$), respectively. As the right panel of Figure 3 indicates, it is attractive for Candidate 0 to adopt the majority-inefficient policy $(0, 0, 0)$. Now, each candidate receives the vote of those citizens in the neighboring nodes. Thus, Candidate 0 wins swing voters $(0, 1, 0)$ (which is less than half of Candidate 1's swing voters) and loses swing voters $(1, 0, 1)$ (which is more than half of his), but the net gain of 9.6% is sufficient for Candidate 0 to win the election.

More generally, one can check that Candidate 0 wins votes by adopting the majority-inefficient position 0 as long as $\mu(\tau|\theta_3 = 1) \in (1/2, 7/9)$. The additional votes are sufficient to swing the election in Candidate 0's favor as long as $\mu(\tau|\theta_3 = 1) \in (1/2, 16/27)$. In this case, candidates play effectively a matching pennies game in which Candidate 1 (0) wins if the candidates choose the same (different, respectively) positions. Thus, both candidates choose the majority-inefficient position with probability 1/2, and the election winner implements the majority-inefficient position with probability 1/2. ■

While Example 2, like the rest of the paper, considers binary positions on each issue, the fundamental reason why it may be optimal for a candidate to choose a majority-inefficient position is robust in more general settings. Consider, for example, a setup where the position on the flexible issue can be chosen from a continuum, such that the median position is majority-efficient. Suppose that, if both candidates choose the same position on the flexible issue, then the number of voters who barely prefer Candidate 1 is larger than the number of voters who barely prefer Candidate 0. Furthermore, suppose that the preferred position on the flexible issue is independently distributed of the voter's net preference for one of the candidates that arises from their fixed positions. In such a situation, Candidate 0 has the same incentive as in Example 2 to deviate from the majority-efficient position, since he can attract a larger number of Candidate 1's swing voters than he loses of his own swing voters.

Proposition 4 also demonstrates clear differences between the PVM and the binary policy model. In the PVM, candidates choose the same policy in equilibrium, and the only reason for why candidates may "cater" to particular groups (more than corresponds to these groups' population weights) is that they may care more about a particular issue and hence are electorally more responsive than the general population. In contrast, all voters in the binary policy model have the same issue weights in their utility functions. Thus, the result that it may be optimal for a candidate to cater to the minority is based on a different reasoning. Also, an equilibrium in which one candidate has an incentive to cater to a minority is in mixed strategies, and therefore candidates' proposed policies diverge with probability 1/2.

The multidimensional structure of our model is crucial for the potential optimality of majority-inefficient positions for candidates. In the one-dimensional standard model, there is only one group of swing voters (i.e., those at or close to the median of the distribution). Note that this is true even if candidates are constrained to choose their position only from a subset of the policy interval. Candidates have to deliver a policy that is popular with the median voter. Furthermore, a policy that the median voter likes is also preferred by a majority of the population. Therefore, in the equilibrium of the standard model, candidates choose majority-efficient policies in order to maximize their chance of winning. This leads to a presumption that candidates should pick popular positions in order to maximize their probability of winning, but this argument logically only applies in a one-dimensional framework.

There is a large literature that tries to explain, within the Downsian model, the empirical observation that candidates often propose considerably divergent policies. Candidates may diverge even though this decreases their winning probability, because they care about the implemented policy (see, e.g., Wittman (1983), Calvert (1985), Roemer (1994), Martinelli (2001), Gul and Pesendorfer (2009)). Other papers obtain policy divergence with office-motivated candidates, but assume incomplete information among voters about candidates characteristics (e.g. Callander (2008)) or among candidates about the position of the median voter (Aragones and Palfrey (2002), Castanheira (2003), Bernhardt, Duggan, and Squintani (2006)).[12] In contrast to all previous papers, policy divergence can arise in the binary policy model in a full information environment, and, unlike in models with policy-motivated candidates, divergence increases a candidate's probability of winning.

4 Plurality versus runoff elections

4.1 Motivation

In this section, we depart from the two-candidate setting in order to provide another application in which the binary policy model provides novel results. When three or more candidates run for election, the problem arises that the Condorcet winner (i.e., the candidate who is preferred by a majority against any opponent) may not win the election. While "third party candidates" (i.e., in the U.S., candidates neither belonging to the Democratic nor the Republican party) often attract only a small number of votes, they can still affect the election outcome. For example, in the 1992, 1996 and 2000 U.S. presidential elections, the election winner did not receive an absolute majority of the votes cast, indicating the importance of votes for third party candidates. This has created concern that "spoiler candidates" can, in general, change the election outcome under plurality rule away from the Condorcet winner.

A number of alternative electoral systems have been proposed to deal with this perceived problem of plurality rule. For example, several local jurisdictions in the U.S. have switched to "instant runoff voting" (IRV) as electoral system for municipal elections (e.g. Minneapolis, San Francisco, Oakland).[13] Another electoral system that has many supporters in the academic community is approval voting.[14]

Our analysis differs in two respects from the existing literature: First, virtually all comparisons of alternative voting institutions and the effects of third-party candidates are set within a one-dimensional framework. Second, candidates are either assumed to be able to commit to any position, or not at all. In contrast, we can analyze both the effect of the electoral system on the election winner, and the effect of a third party candidate on the endogenous part of the platforms of major party candidates. This new endogenous effect can be of crucial importance as it can overturn the standard welfare comparison of runoff versus plurality electoral systems.

[12] A third branch of literature, which is less directly related to this paper, explains policy divergence as entry deterrence by two dominant parties (e.g., Palfrey (1984), Callander (2005)).

[13] IRV is a voting system for single winner elections in which voters rank candidates in order of preference. If no candidate receives an overall majority of first preferences in an IRV election, the candidate with the fewest votes is eliminated, and his votes are transferred according to his voters' second preference. All votes are retallied, and the procedure is repeated until one candidate achieves a majority.

[14] See, e.g., Brams and Fishburn (1978), Cox (1987), Myerson and Weber (1993). Under approval voting, each voter is free to vote for as many candidates as he chooses. The candidate with the most votes is elected. The intuition for the advantage of approval voting is as follows: Under plurality rule, voters may ignore a moderate candidate and focus on two extreme candidates, if they fear that the moderate has no chance of winning. In contrast, under approval voting, voters can vote for *both* their preferred extreme candidate and the moderate candidate, so that they don't have to fear to waste their vote. This should make it easier for moderate candidates to win.

4.2 Plurality vs. runoff when all positions or no positions can be chosen

As a benchmark for our analysis, we start with the Downsian assumption that candidates can choose all of their positions (or, equivalently, that all candidates have the same fixed positions). In all of the following, we assume that voters vote "sincerely", i.e., for their most preferred candidate. If voters are indifferent between several candidates, then they are equally likely to cast their vote for each of them. In elections with three or more candidates, there are usually many Nash equilibria in undominated strategies. However, the equilibrium in which voters vote sincerely is a natural focal point.[15] Note that Propositions 5 and 6 in this section hold for any general voter preferences, not just when voters have weighted-issue preferences.

Proposition 5 *Suppose that there are n candidates with the same choice set:* $A^0 = A^1 = \cdots = A^n$*. Assume that there exists a unique majority-efficient position* $a^* \in A^0$*.*

1. *Under runoff rule, there exists an equilibrium in which all candidates choose* a^**.*
2. *Let* $\tilde{A}$ *be the set of all policies that are preferred to* a^* *by more than* $1/n$ *of the voters. If* $\tilde{A}$ *is non-empty, then there is no equilibrium under plurality rule such that all candidates always play* a^**, and the probability that a candidate with a majority-inefficient position wins the election is strictly positive.*

When all opponents choose a^* under runoff rule, then the best response is to play a^* as well: Clearly, there is no majority-inefficient policy with which a candidate could win an outright majority in the first round, and to have a chance of winning in the second round against an opponent who chooses a^*, a candidate has to choose a^* as well.

In contrast, there is usually no equilibrium under plurality rule in which all candidates choose majority-efficient positions. If all $n-1$ opponents choose the majority-efficient position, then a candidate can win for sure by playing some element of $\tilde{A}$. Thus, there is no pure strategy equilibrium in which all candidates play a^*, and as a consequence, the winning position under plurality rule is majority-inefficient with a strictly positive probability.

As an example, suppose that there are three candidates and only one issue, and that a proportion $p \in (1/2, 2/3)$ prefers position 0, while the remainder of the electorate prefers position 1. Under runoff rule, the unique equilibrium is that all three candidates choose position 0. Under plurality rule, a candidate is guaranteed to win if both opponents take the opposite position, and each candidate wins with probability $1/3$ if all candidates take the same position. Clearly, there is no pure strategy equilibrium. From the symmetry of the two strategies with respect to the candidates' winning chance, it is easy to see that each candidate randomizes between both policies with probability $1/2$ each in the unique mixed strategy equilibrium. Consequently, the probability that the majority-inefficient policy 1 wins is $1/2$ in equilibrium.

[15] Degan and Merlo (2006) suggest that, empirically, most observed voting behavior is consistent with the assumption that voters vote sincerely.

We now turn to the case that politicians differ in their fixed positions and cannot choose positions on any issue. Again, runoff rule leads to (weakly) better results than plurality rule in this setup.

Proposition 6 *Let a^j denote the entirely fixed position of Candidate j, and suppose that there is a Condorcet loser a^n (i.e., Candidate n would lose a two-way race against any other candidate).*[16] *Under plurality rule, the election winner may be any policy, while a^n is not a possible election outcome under runoff rule. Also, the election outcome under runoff rule, a_R, is weakly majority preferred to the election outcome under plurality rule, a_P.*

The Condorcet loser a^n can certainly have the most first preferences in the electorate (and hence win under plurality rule), but cannot receive an outright majority in the first round of a runoff system, and loses the runoff against any opponent (hence, cannot win in a runoff system). The intuition for why a_R is always (at least weakly) preferred to a_P is that the plurality rule winner is at least guaranteed to proceed to the second round in a runoff system; thus, for a majority of voters, the outcome under runoff is at least as good as the plurality outcome.

4.3 Runoff versus plurality with some fixed and some flexible positions

Together, the results of the previous section show that runoff rule weakly dominates plurality rule in terms of electoral outcome, if candidates can either choose *all* of their positions, or *none* of them. It is therefore tempting to conclude that runoff rule is generally (at least weakly) better for society than plurality rule. However, our results so far warn against drawing such a conclusion prematurely. Indeed, we now show that, if candidates differ in their sets of feasible policies, then qualitatively different results can arise in our model. The intuition is that the set of relevant swing voters differs between plurality rule and runoff rule. In Example 3, the set of swing voters is smaller under plurality rule than under runoff rule, but it is nevertheless more representative for the population at large.

The example is a somewhat more elaborate version of Example 1, in which the two candidates cater to the minority, because minority types are more willing to change their voting behavior than majority types. In other words, *among swing voters*, the (overall) minority is the majority. We then introduce a third candidate who has no chance of winning, but changes the composition of swing voters who are relevant for the two main candidates in a way that now the overall majority in the population is also the majority of swing voters; thus, under plurality rule, the main candidates now choose the majority-efficient policy. In contrast, under runoff rule, the main candidates essentially ignore the third candidate because they care only about their showdown against each other in the second round, after the third candidate is eliminated, and the equilibrium has the same inefficient features of Example 1.

Example 3 Candidate 0 is fixed to $(0, 0)$, and Candidate 1 is fixed to $(1, 0)$, on the first two issues. Both these candidates can freely choose their position on the third issue. Candidate 2 is fixed at $(1, 1, 1)$. The following Table 1 gives the proportions and issue weights of all voter types.

[16] This assumption is clearly satisfied if there is a transitive majority preference ranking of candidates $a^0 \succ^* a^1 \succ^* \ldots \succ^* a^n$, but our assumption is more general.

Proportion	Preferred policy	λ_1	λ_2	λ_3
26%	$(0, 0, 0)$	2	5	1
24%	$(1, 0, 0)$	2	5	1
10%	$(0, 0, 0)$	1	5	2
10%	$(1, 0, 0)$	1	5	2
6%	$(0, 0, 1)$	1	5	2
6%	$(1, 0, 1)$	1	5	2
9%	$(0, 1, 1)$	1	5	2
7%	$(1, 1, 1)$	1	5	2
2%	$(1, 1, 0)$	1	5	2

Table 1: Voter distribution

Note that 82% of the population strongly dislike Candidate 2's position on the second issue and issue two is very important compared to the other issues. Thus, Candidate 2 is truly a "spoiler" who has no hope of actually winning the election: Only 18% of the population would ever vote for Candidate 2, so he can neither be the top vote getter in a plurality election nor win the first or second round in a runoff system. On the third issue, 72% of the population prefer policy 0 while 28% prefer policy 1, so that policy 0 is majority-efficient (for both Candidate 0 and 1).

If only the first two candidates stand for election, the first two voter types will always vote for their respective candidate, independent of the positions that candidates take on issue 3; in contrast, the remaining voters are the potential swing voters between Candidates 0 and 1. Note also that, among swing voters, a majority (28% versus 22%) prefers policy 1 on the third issue. Depending on the policies that the candidates choose, the vote shares are given in Table 2. Note that a candidate's vote share is always 3 percentage points higher when he chooses $a_3 = 1$. Thus, in vote shares, policy $a_3 = 1$ is strictly dominant, and it is weakly dominant in terms of the winning probability. In the equilibrium in weakly dominant strategies, both candidates therefore choose policy $a_3 = 1$.[17]

	$(1, 0, 0)$	$(1, 0, 1)$
$(0, 0, 0)$	51%, 49%	48%, 52%
$(0, 0, 1)$	54%, 46%	51%, 49%

Table 2: Candidates' vote shares

Under runoff rule with three candidates, the same logic applies for the policy choice of Candidates 0 and 1: Both are guaranteed to proceed into the second round, because each has a core support (from one of the first two types) that is larger than the 18% that Candidate 2 gets from the last three preference groups. Thus, in their policy choice under runoff rule, Candidates 0 and 1 face exactly the same problem

[17]Since candidate 1 always loses, $a_3^0 = 1$ and $a_3^1 = 0$ is also a Nash equilibrium. Because the implemented policy is the same as in the equilibrium in weakly dominant strategies, the welfare comparison between plurality and runoff rule would be the same as in the text.

as if Candidate 2 did not exist, and therefore will choose the same positions as in the two-candidate election above: Thus, $a_3^0 = 1$ and $a_3^1 = 1$ under runoff rule, and Candidate 0 wins with policy $(0, 0, 1)$.

Now consider what happens under plurality rule in a three candidate election: Candidate 2 attracts the votes of the last three voter types and effectively removes them from the set of swing voters who are relevant for Candidates 0 and 1. Choosing policy 1 instead of 0 on the third issue would now only attract 6%, but, at the same time, lose 10%. The unique equilibrium in undominated strategies has $a_3^0 = 0$ and $a_3^1 = 0$ under plurality rule, and Candidate 0 wins with policy $(0, 0, 0)$.

The implemented policy under plurality rule with three candidates, $(0, 0, 0)$, is majority-preferred to $(0, 0, 1)$, which is both the equilibrium policy when there are only two candidates and the equilibrium policy under runoff rule. Thus, Example 3 shows that plurality rule can lead to better results than runoff rule, and that the existence of a spoiler who cannot win can improve welfare for a majority of voters.[18]

■

There is certainly no guarantee that plurality rule is better than runoff rule (in the sense of majority-efficiency) in the case that candidates differ in their fixed positions. Indeed, it is simple to adjust the examples given in the last subsection to include some trivial fixed differences between candidates, and runoff rule would still generate better results than plurality rule. However, Example 3 demonstrates that plurality rule *may* (in a robust example) be strictly better than runoff rule when candidates differ in their fixed positions and can choose a position on some other issues. As the results of the previous subsection show, this result cannot be obtained if candidates can choose all positions, or no position at all. Since these two cases are the only ones that can arise in a one-dimensional framework, a result such as Example 3 requires a multidimensional setup.

While we have focused our comparison of electoral systems on plurality rule and runoff rule, the fundamental insight we obtain applies more generally. Suppose that there is an electoral system, call it system A, that always selects the Condorcet winner from any given set of candidates. In Example 3, system A corresponds to runoff rule, but this could also be approval voting with strategic voters and some refinement, say the "voting equilibrium" concept of Myerson and Weber (1993), or other electoral systems suggested in the literature. Now suppose that we compare the efficiency of electoral system A with electoral system B (plurality rule in our example) that does not always select the Condorcet winner from a given set of candidates. If the positions of candidates are fixed in all issues, or if candidates are flexible in all issues, then system A is indeed better than system B, in the sense that, if the two systems produce different outcomes, then a majority of voters prefers the outcome under electoral system A. However, as Example 3 shows, this is not anymore true, if some positions are fixed and others are flexible for the candidates. Our result thus shows that a search for "the best" electoral system (in the sense of selecting the Condorcet winner for the largest possible set of voter preference profiles) is not necessarily a useful objective: Even if we were to find such an optimal electoral system for fixed

[18]Note that, under plurality rule, the spoiler candidate likes the effect of his entry on the policy that is implemented: The identity of the winning candidate is unchanged, but the winner proposes a policy on the third issue that the spoiler prefers relative to the policy that would be proposed, if there are only two candidates. Thus, Example 3 is robust if we endogenize the entry decision of the spoiler candidate, provided that this candidate's policy motivation is sufficiently large in comparison to the cost of running.

positions, the outcome under this system might be dominated by the outcome under plurality rule (or, some other “non-optimal” system).

4.4 Related literature on multicandidate elections

The effect of third party candidates has previously been analyzed in the Downsian model and the citizen candidate framework. In a Downsian model with three candidates, no pure strategy equilibrium exists when candidates choose simultaneously, assuming that voters vote sincerely.[19] Thus, many models assume some exogenous distinction between candidates to obtain pure strategy equilibria. In Palfrey (1984) and Callander (2005), one candidate (who is interpreted as the “third party candidate”) chooses his platform after the two “main” candidates. In Palfrey, the threat of entry by the third party candidate forces the two main candidates to choose positions that are equidistant from the median. The third party candidate (who is supposed to maximize his vote share if he cannot win) chooses a more extreme position than either candidate, and one that is as close as possible to one of the two main candidates, and loses for sure. He also induces the loss of the candidate next to whom he chooses to position himself. Thus, in this framework, policy-motivated third party candidates would either run on a platform opposite to what they really prefer (if they can commit), or not run at all (if they cannot commit).

Osborne and Slivinski (1996) analyze the issue of third party candidates in a citizen candidate model, and also compare plurality rule and runoff rule voting systems. The set of equilibrium positions in two candidate races is more moderate under runoff rule than under plurality rule, so that, from the point of view of the median voter and the majority of the population, runoff rule is a better electoral system than plurality rule. Under plurality rule, there can be equilibria with a spoiler candidate who enters the election in spite of having no chance of winning. This happens if the spoiler candidate is located between the two main candidates and draws more votes from the candidate who is farther away from the spoiler (this is possible only if the distribution of voters is asymmetric).

5 Conclusion

The binary policy model provides an intuitive and tractable framework for the analysis of multidimensional policy choice. The model allows us to study what happens when candidates’ positions are fixed in some dimensions (possibly to different policies), while they can commit on other issues. This combination of the Downsian model and the citizen candidate model, two central models in the literature, is both realistic and yields truly novel results.

The most interesting of our results arise from the interplay of multidimensionality and candidates’ differentiated fixed positions (or characteristics) on some issues. Voter preferences for fixed positions imply that some voters will vote for one of the candidates irrespective of the positions of candidates on flexible issues. Candidates only compete for the support of the remaining “swing voters”. We identify

[19] To our knowledge, the entire literature that compares the effects of electoral systems assumes sincere voting because voting equilibria in plurality elections with more than two candidates abound (and cannot be significantly narrowed down through standard refinements). Therefore no conclusions could be reached with the assumption of strategic voters. Also note that sincere voting is a Nash equilibrium in undominated strategies under plurality rule.

two distinct reasons for why candidates may choose minority-preferred positions in equilibrium. First, the preference distribution on flexible issues among swing voters may differ substantially from the preference distribution in the population at-large. Second, one candidate may have fewer swing voters to "defend" than his opponent, and thus may benefit by differentiating from his opponent, even if his opponent takes a position that is popular with a majority of swing voters.

Thus, in our framework, policy divergence can arise with two office-motivated candidates and no uncertainty about the distribution of voters. This implies that two standard results of the Downsian model — policy convergence of candidates, and movement of candidates "into the middle", i.e., in a direction that is preferred by a majority of the electorate — are actually generated by the sameness of candidates in, and the one-dimensional structure of, the Downsian model.

Our focus on binary positions in each issue simplifies the model, in particular the description of voter preferences.[20] However, it is intuitive that the main insights from our binary model would continue to hold. One could certainly study the effect of fixed issues in a Euclidean framework with one dimension in which candidates are flexible. For a given location of the opponent on the flexible issue, a candidate may not necessarily have an incentive to locate close to his opponent: The reason is that the swing voters are not necessarily located (only) between the positions of the two candidates, as voters have preferences over the candidates' other, fixed dimensions. In fact, our examples of policy divergence from the binary model can easily be embedded in the continuum model. However, a characterization of equilibria of the continuum would be very challenging, as pure strategy equilibria only exist in special cases. Analyzing mixed strategy equilibria would again require moving to a discrete setting.

We also apply our model to analyze elections with more than two candidates. In this case, runoff rule – or any other rule that selects the Condorcet winner more often than plurality rule – weakly majority dominates plurality rule, if candidates either cannot commit at all (the citizen-candidate case), or are completely flexible on all dimensions (the Downsian case). These results, while new in the binary policy framework, mirror intuitions in the previous literature. However, when candidates have some fixed positions and are flexible in the remaining issues, then the opposite case may arise: Even though runoff rule (in our framework) always selects the Condorcet winner from a *given* set of candidates, and plurality rule does not, the effect of the electoral system on the policies that candidates propose can be such that a majority strictly prefers the equilibrium outcome under plurality rule than under runoff rule. This result casts doubt on whether there is an "optimal" electoral system (or even just one that is always "better" than plurality rule in the sense of majority-preference) for a large set of preferences.

[20]For example, if an issue has three possible positions, then, in order to describe a voter's preference, we need to specify a ranking of these positions, as well as how much the voter likes his second-most preferred position relative to his top and least-preferred choice. The more possible positions there are, the more complicated this becomes.

6 Appendix

Proof of Remark 1. The "only if" direction is obvious. Suppose that the "if" statement is false. Since a^0 is majority-efficient, there must be $\tilde{a} \in A^1$ such that $\tilde{a} \succ^* a^0$. But then, Candidate 1 could win by playing $\tilde{a}$, which cannot be true in equilibrium. ■

Proof of Lemma 1. Let $a^j \in A^j$ be arbitrary. Then $\overline{a}^j \succeq a^j$ if and only if at least 50% of the population prefers $\overline{a}^j$ to a^j, i.e.,

$$\mu\left(\left\{\theta \,\Big|\, \sum_{i=1}^{I} \lambda_i |\theta_i - \overline{a}^j| \leq \sum_{i=1}^{I} \lambda_i |\theta_i - a^j|\right\}\right) \geq \mu\left(\left\{\theta \,\Big|\, \sum_{i=1}^{I} \lambda_i |\theta_i - \overline{a}^j| \geq \sum_{i=1}^{I} \lambda_i |\theta_i - a^j|\right\}\right) \tag{7}$$

Let $D = \{i | a^j \neq \overline{a}^j\}$. Then $D \subset S^j$. Using the definition of X_i implies that (7) is equivalent to

$$\mu\left(\left\{\theta \,\Big|\, \sum_{i \in D}^{I} \lambda_i X_i \geq \sum_{i \in D} \lambda_i (1 - X_i)\right\}\right) \geq \mu\left(\left\{\theta \,\Big|\, \sum_{i \in D}^{I} \lambda_i X_i \leq \sum_{i \in D} \lambda_i (1 - X_i)\right\}\right), \tag{8}$$

which in turn is equivalent to

$$\mu\left(\left\{\theta \,\Big|\, \sum_{i \in D}^{I} \lambda_i X_i \geq 0.5 \sum_{i \in D} \lambda_i\right\}\right) \geq \mu\left(\left\{\theta \,\Big|\, \sum_{i \in D}^{I} \lambda_i X_i \leq 0.5 \sum_{i \in D} \lambda_i\right\}\right). \tag{9}$$

Since $\mu\left(\left\{\theta \,\Big|\, \sum_{i \in D}^{I} \lambda_i X_i \geq 0.5 \sum_{i \in D} \lambda_i\right\}\right) + \mu\left(\left\{\theta \,\Big|\, \sum_{i \in D}^{I} \lambda_i X_i \leq 0.5 \sum_{i \in D} \lambda_i\right\}\right) \geq 1$, inequality (9) implies $\mu\left(\left\{\theta \,\Big|\, \sum_{i \in D}^{I} \lambda_i X_i \geq 0.5 \sum_{i \in D} \lambda_i\right\}\right) \geq 0.5$ and hence

$$\text{median}\left(\sum_{i \in D} \lambda_i X_i\right) \geq 0.5 \sum_{i \in D} \lambda_i. \tag{10}$$

Conversely, if (10) is satisfied, then the left-hand side of (9) is at least 0.5, while the right-hand side is at most 0.5, and hence (9) holds.

Because we have shown equivalence between (7) and (10) it also follows that if (10) holds for all D then (7) holds for all $a^j \in A^j$, i.e., $\overline{a}^j$ is then majority-efficient. ■

Proof of Corollary 1. See text. ■

Proof of Proposition 1. We first prove that $\overline{a}^j$ is majority-efficient if (2) holds.

Let Y_i be a collection of i.i.d random variable, each of which assumes values 0 and 1 with probability 0.5. Let G be the distribution of $Y = (Y_1, \ldots, Y_I)$. Note that $G(x) = \prod_{i \in S^j} 0.5^{1-x_i}$. Thus, (2) implies that F first order stochastically dominates G, i.e., $F(x) \leq G(x)$ for all $x \in \{0, 1\}^I$. As indicated in Tarp and Osterdal (2007) it is immediate that one can derive F from G by iteratively moving probability mass to higher realizations, i.e., from values x to $x' \geq x$. Thus, there exists a multidimensional random variable $Z = (Z_1, \ldots, Z_I) \geq 0$ such that $X = Y + Z$. Therefore,

$$\text{median}\left(\sum_{i\in D} \lambda_i X_i\right) \geq \text{median}\left(\sum_{i\in D} \lambda_i Y_i\right). \tag{11}$$

Next, note that the distribution of $\sum_{i\in D} \lambda_i Y_i$ is symmetric, and thus

$$\text{median}\left(\sum_{i\in D} \lambda_i Y_i\right) = E\left[\sum_{i\in D} \lambda_i Y_i\right] = 0.5 \sum_{i\in D} \lambda_i. \tag{12}$$

(11) and (12) imply that the condition of Lemma 1 is satisfied. Hence, $\overline{a}^j$ is majority-efficient.

Now suppose there exists $x \in \{0,1\}^I$ such that

$$F(x) > \prod_{i\in S^j} 0.5^{1-x_i}. \tag{13}$$

Let x_S be the set of selectable issues. Then (13) implies $x_S \neq (1, \ldots, 1)$. Let $I(x) = \{i \in S^j | x_i = 0\}$. If $I(x)$ consists of a single issue, then we are done. In particular, let $x_k = 0$ and $x_i = 1$ for all $i \neq k$. Let $\hat{a}^j$ be the policy that we get from $\overline{a}^j$ if we replace the position k by its opposite, i.e., $\hat{a}^j_k \neq \overline{a}^j_k$ and $\hat{a}^j_i = \overline{a}^j_i$ for all $i \neq k$. Then (13) implies $F(x) > 1/2$. However, since $F(x) = \mu(\{\theta_k \neq \overline{a}^j_k\})$, this implies that $\hat{a}^j$ is majority preferred to $\overline{a}^j$ and $\overline{a}^j$ is therefore not majority-efficient.

We now proceed by induction on the size of $I(x)$. In particular, suppose we have already shown that if $\#I(x) < m$ and (13) holds then there exists $\hat{a}^j$ that is majority preferred to $\overline{a}^j$. Suppose that $\#I(x) = m$, i.e., $I(x) = \{i_0, \ldots, i_{m-1}\}$. Then by assumption

$$\mu\left(\{\theta_{i_0} \neq \overline{a}^j_{i_0}\} \,\middle|\, \{\theta_{i_1} \neq \overline{a}^j_{i_1}, \ldots \theta_{i_{m-1}} \neq \overline{a}^j_{i_{m-1}}\}\right) \leq \mu\left(\{\theta_{i_0} \neq \overline{a}^j_{i_0}\}\right), \tag{14}$$

and (13) implies

$$\mu\left(\{\theta_{i_0} \neq \overline{a}^j_{i_0}, \theta_{i_1} \neq \overline{a}^j_{i_1}, \ldots \theta_{i_{m-1}} \neq \overline{a}^j_{i_{m-1}}\}\right) > (1/2)^m. \tag{15}$$

(14) and (15) imply that either $\mu\left(\{\theta_{i_1} \neq \overline{a}^j_{i_1}, \ldots \theta_{i_{m-1}} \neq \overline{a}^j_{i_{m-1}}\}\right) > (1/2)^{m-1}$ or $\mu\left(\{\theta_{i_0} \neq \overline{a}^j_{i_0}\}\right) > (1/2)$. Let $x', x'' \in \{0,1\}^{S^j}$ be defined by $x'_i = 1$ if and only if $i \in \{i_1, \ldots, i_{m-1}\}$ and $x''_{i_0} = 0$, $x''_i = 1$ for all $i \neq i_0$. Then (13) holds either for x' or x''. However, since $\#I(x'), \#I(x'') < m$ the induction hypothesis implies that there exists $\hat{a}^j$ that is majority preferred to $\overline{a}^j$. Hence $\overline{a}^j$ is not majority-efficient. ■

Proof of Corollary 2. See text. ■

Proof of Proposition 2. Suppose that (a^0, a^1) is an equilibrium. Then, each candidate wins with probability 0.5. (Suppose, to the contrary, that Candidate 0 (say), always loses. However, because $A^0 = A^1$, he could improve by choosing $\tilde{a}^0 = a^1$, a contradiction). Let $\hat{a}$ be an arbitrary feasible policy. If $\hat{a} \succ^* a^0$, then Candidate 1 wins (with probability 1) if he offers policy $\hat{a}$. Since a^0 is an equilibrium strategy, $a^0 \succeq^* \hat{a}$ for all $\hat{a}$. Similarly, $a^1 \succeq^* \hat{a}$ for all $\hat{a}$. Hence, both a^0 and a^1 are majority-efficient.

Now suppose that a^0 and a^1 are majority-efficient. We have to show that (a^0, a^1) is an equilibrium. Since $a^0 \geq^* a^1$ and $a^1 \geq^* a^0$ (by majority-efficiency), each candidate gets 50% of the votes and thus wins with probability 0.5. Furthermore, by majority-efficiency of a^0 and a^1, $a^0 \geq^* \hat{a}$ and $a^1 \geq^* \hat{a}$, for all $\hat{a}$. Hence, there is no profitable deviation, so that (a^0, a^1) is an equilibrium.

Now consider a mixed strategy equilibrium (σ^0, σ^1). Each candidate must win with probability 0.5 (otherwise, the candidate who wins with the lower probability could deviate to the strategy of his opponent, thereby increasing his winning probability to 0.5). Furthermore, in order for mixing to be optimal, every policy in the support of σ^j must give agent j a winning probability of 0.5. Now, assume by way of contradiction that the support of σ^j contains a set B of policies that are not majority-efficient. Because the set of policies is finite, B must occur with strictly positive probability. Then policies in B only win if Candidate $-j$ also selects a non-majority-efficient policy. Because the winning probability must be 0.5, this implies that the opponent uses a non-majority-efficient strategy with strictly positive probability. Let $\tilde{a}^j$ be a majority-efficient policy. Suppose that Candidate j uses the alternative strategy $\tilde{\sigma}^j$ which uses $\tilde{a}^j$ whenever a policy in B is selected under σ^j and corresponds to σ^j, otherwise. Then $\tilde{a}^j$ wins whenever the opponent selects a non-majority-efficient policy and ties whenever the opponent uses a majority-efficient policy. Thus, Candidate j's winning probability strictly increases, a contradiction. Hence, every policy in the support of σ^j is majority-efficient. ■

Proof of Proposition 3. For $(0,0)$ to be an equilibrium, it must be true that

$$\begin{aligned} \mathrm{SV}_0 &\geq (1-\xi_0)\mathrm{SV}_0 + (1-\xi_1)\mathrm{SV}_1 \text{ and} \\ \mathrm{SV}_1 &\geq (1-\xi_0)\mathrm{SV}_0 + (1-\xi_1)\mathrm{SV}_1, \end{aligned}$$

which can be rearranged to give (4). Similarly, for $(1,1)$ to be an equilibrium, it must be true that

$$\begin{aligned} \mathrm{SV}_0 &\geq \xi_0\mathrm{SV}_0 + \xi_1\mathrm{SV}_1 \text{ and} \\ \mathrm{SV}_1 &\geq \xi_0\mathrm{SV}_0 + \xi_1\mathrm{SV}_1, \end{aligned}$$

which is equivalent to (5). For $(0,1)$ to be an equilibrium, it must be true that

$$\begin{aligned} \mathrm{SV}_0 &\leq \xi_0\mathrm{SV}_0 + \xi_1\mathrm{SV}_1 \text{ and} \\ \mathrm{SV}_1 &\leq (1-\xi_0)\mathrm{SV}_0 + (1-\xi_1)\mathrm{SV}_1. \end{aligned}$$

This implies $\frac{\mathrm{SV}_1}{\mathrm{SV}_0} = \frac{\xi_0}{1-\xi_1} = \frac{1-\xi_0}{\xi_1}$. Cross-multiplying the last equality implies $\xi_0 + \xi_1 = 1$, and using this implies $\frac{\mathrm{SV}_1}{\mathrm{SV}_0} = 1$. ■

Proof of Proposition 4. First, note that we can renumber issues such that $\{1, \ldots, m\}$ is the set of fixed issues. Further, we can assume without loss of generality that $\mu(\{\tau|\theta_i = a_i^0\}) > 0.5$ for all $i \in \{1, \ldots, k\}$, $\mu(\{\tau|\theta_i = a_i^1\}) \geq 0.5$ for all $i \in \{k+1, \ldots, k'\}$, and that $a_i^0 = a_i^1$ for all $i \in \{k'+1, \ldots, m\}$. Since ϕ is one-to-one it follows that $k' \leq 2k$. We first assume that $k' = 2k$.

For all $i \in \Im$, let $\overline{a}_i = 1$ if $\mu(\{\tau|\theta_i = 1\}) > 0.5$ and $\overline{a}_i = 0$ if $\mu(\{\tau|\theta_i = 1\}) < 0.5$. Define the random variable X_i by

$$X_i(\theta) = \begin{cases} 1 & \text{if } \theta_i = \overline{a}_i \\ 0 & \text{if } \theta_i \neq \overline{a}_i \end{cases}$$

Note that voters of type θ strictly prefer Candidate 0 or are indifferent between the candidates if

$$\sum_{i=1}^{k} \lambda_i X_i(\theta) + \sum_{i=k+1}^{2k} \lambda_i(1 - X_i(\theta)) + \sum_{k=m+1}^{I} \lambda_i X_i(\theta) \geq \sum_{i=1}^{k} \lambda_i(1 - X_i(\theta)) + \sum_{i=k+1}^{2k} \lambda_i X_i(\theta) + \sum_{k=m+1}^{I} \lambda_i(1 - X_i(\theta)),$$

which is equivalent to

$$\sum_{i=1}^{k}(\lambda_i X_i(\theta) - \lambda_{i+k} X_{i+k}(\theta)) + \sum_{k=m+1}^{I} \lambda_i X_i(\theta) \geq 0.5 \sum_{i=1}^{k}(\lambda_i - \lambda_{i+k}) + 0.5 \sum_{k=m+1}^{I} \lambda_i. \tag{16}$$

Let $p_i = \mu(\{\theta_i = \overline{a}_i\})$. Define $\hat{X}_i = \lambda_i X_i - \lambda_{i+k} X_{i+k}$. Then $\hat{X}_i$ has the four realizations $-\lambda_{i+k}$, 0, $\lambda_i - \lambda_{i+k}$, and λ_i, which occur with probabilities $(1 - p_i)p_{i+k}$, $(1 - p_i)(1 - p_{i+k})$, $p_i p_{i+k}$, and $p_i(1 - p_{i+k})$. Let $\hat{Y}_i$ be a random variable which has realizations $-\lambda_{i+k}$ and λ_i with the same probability of $q = 0.5((1 - p_i)p_{i+k} + p_i(1 - p_{i+k}))$, and the remaining two realizations 0 and $\lambda_i - \lambda_{i+k}$ with the same probability of $q' = 0.5((1 - p_i)(1 - p_{i+k}) + p_i p_{i+k})$. Since $p_i > p_{i+k}$ it follows immediately that $\hat{X}_i$ first order stochastically dominates $\hat{Y}_i$. Thus, there exists a random variable $\hat{Z}_i \geq 0$ such that $\hat{X}_i = \hat{Y}_i + \hat{Z}_i$. Furthermore, note that $E[\hat{Y}] = (q + q')(\lambda_i - \lambda_{i+k}) = 0.5(\lambda_i - \lambda_{i+k})$.

Next, note that X_i first order stochastically dominates a random variable Y_i which pays 1 with probability 0.5 and 0 with probability 0.5. Thus, for $i > m$ we can find random variables Z_i with $Z_i \geq 0$ and $X_i = Y_i + Z_i$. Clearly, $E[Y_i] = 0.5\lambda_i$ Thus,

$$\begin{aligned}
&\mu\left(\left\{\tau \,\Big|\, \sum_{i=1}^{k}(\lambda_i X_i(\theta) - \lambda_{i+k} X_{i+k}(\theta)) + \sum_{k=m+1}^{I} \lambda_i X_i(\theta) \geq 0.5 \sum_{i=1}^{k}(\lambda_i - \lambda_{i+k}) + 0.5 \sum_{k=m+1}^{I} \lambda_i\right\}\right) \\
&= \mu\left(\left\{\tau \,\Big|\, \sum_{i=1}^{k}(\hat{Y}_i(\theta) + \hat{Z}_i(\theta)) + \sum_{k=m+1}^{I} \lambda_i(Y_i(\theta) + Z_i(\theta) \geq \sum_{i=1}^{k} E[\hat{Y}_i] + \sum_{k=m+1}^{I} E[Y_i]\right\}\right) \\
&> \mu\left(\left\{\tau \,\Big|\, \sum_{i=1}^{k} \hat{Y}_i(\theta) + \sum_{k=m+1}^{I} \lambda_i Y_i(\theta) \geq \sum_{i=1}^{k} E[\hat{Y}_i] + \sum_{k=m+1}^{I} E[Y_i]\right\}\right) = 0.5,
\end{aligned}$$

where the last equality follows because both Y_i and $\hat{Y}_i$ are symmetrically distributed. In view of (16) this means that Candidate 0 wins by receiving more than 50% of the vote share. This proves the first statement for $k' = 2k$.

Now suppose that $k' < 2k$. Let $\underline{p} > 0.5$ such that $\underline{p} < \mu(\{\tau|\theta_i = a_i^0\})$ for all $i < k$. Then define $2k - k'$ artificial issues such that $0.5 < \mu(\{\tau|\theta_i = a^1\}) < \underline{p}$ and $\lambda_i = 0$ for these issues. Now both candidates $j = 0, 1$ have the same number of issues with $\mu(\{\tau|\theta_i = a^j\}) > 0.5$, and as a consequence the first part of the argument implies. This concludes the proof of the first statement.

To prove the second statement, first set $\lambda_i = 0$ for all $i \notin \{1, k + 1, I\}$. We now show that there exist weights λ_1, λ_k and λ_I such that Candidate 1 wins by choosing the majority-inefficient position on issue I. In view of the first part of the proof, we must have $\lambda_1 < \lambda_{1+k}$, else Candidate 0 always wins.

If Candidate 0 chooses the majority-inefficient position on issue I, then in view of (16) voter θ votes for Candidate 0 if

$$\lambda_1 X_1 - \lambda_{k+1} X_k - \lambda_I X_i \geq 0.5(\lambda_1 - \lambda_{1+k} - \lambda_I). \tag{17}$$

If Candidate 1 chooses the same position as Candidate 0 on issue I then voter θ votes for Candidate 0 if

$$\lambda_1 X_1 - \lambda_{k+1} X_k \geq 0.5(\lambda_1 - \lambda_{1+k}). \tag{18}$$

Since $\lambda_1 < \lambda_{k+1}$ it follows that (18) is satisfied only for types θ with $\theta_{i+k} \neq \overline{a}_{i+k}$. Since $\mu(\{\theta_{i+k} \neq \overline{a}_{i+k}) < 0.5$, this implies that Candidate 0 loses.

Now suppose that $\lambda_{k+1} - \lambda_1 < \lambda_I < \lambda_1 + \lambda_{k+1}$. Then (18) is satisfied (with a strict inequality) for all $\theta \in A = \{\theta | \theta_1 = \theta_{k+1} = \theta_I = 0$, or $\theta_1 = 1$, $\theta_{k+1} = \theta_I = 0$, or $\theta_1 = \theta_{k+1} = 1$, $\theta_I = 0$, or $\theta_1 = \theta_I = 1$, $\theta_{k+1} = 0\}$. Clearly,

$$\begin{aligned}\mu(A) = &\mu(\{\theta_1 = a_1^1\})\mu(\{\theta_{k+1} = a_{k+1}^0\}) + \mu(\{\theta_1 = a_1^0\})\mu(\{\theta_{k+1} = a_{k+1}^1\})\mu(\{\theta_I \neq \overline{a}_I\}) \\ &+ \mu(\{\theta_1 = a_1^0\})\mu(\{\theta_{k+1} = a_{k+1}^0\})\mu(\{\theta_I = \overline{a}_I\})\end{aligned}$$

Since $\mu(\{\theta_1 = a_1^0\}) > \mu(\{\theta_{k+1} = a_{k+1}^1\})$, it follows immediately that $\mu(A) > 0.5$ if $\mu(\{\theta_I = \overline{a}_I\})$ is close to 0.5. Thus, there exists a $\varepsilon > 0$ such that Candidate 0 wins by choosing the majority-inefficient position on issue I if $\mu(\{\theta_I = \overline{a}_I\}) < 0.5 + \varepsilon$. The same argument applies for any $i > m$. Finally, note that the candidates' vote share are continuous in a neighborhood of Λ of λ. This proves the second statement of the proposition. ■

Proof of Proposition 5. Suppose that all candidates except for Candidate j play a^*, and that Candidate j plays a^j. Clearly, a^j cannot win outright in the first round. If Candidate j does not proceed to the runoff round, then his payoff is zero. Suppose now that Candidate j enters the runoff against a Candidate i. Since a^* is the unique majority-efficient policy, $\mu(\{\tau | u_\tau(a^*) \geq u_\tau(a^j)\}) > \mu(\{\tau | u_\tau(a^j) \geq u_\tau(a^*)\})$. Hence, Candidate i (who plays a^*) wins the runoff round against Candidate j. Thus, all candidates playing a^* is an equilibrium, as any deviating candidate would always lose.

To show the second statement, note that, if all n candidates play a^* then each wins with probability $1/n$. Now suppose that $\tilde{A} \neq \emptyset$ and Candidate j deviates to some $a^j \tilde{A}$. Since $\mu(\{\tau | u_\tau(a^*) \geq u_\tau(a^j)\}) > 1/n$, each of the remaining $n - 1$ candidates (who, by assumption, split the vote equally) receives less than $1/n$. As a consequence, Candidate j wins. Thus, all candidates playing a^* is not an equilibrium. ■

Proof of Proposition 6. Clearly, a^n cannot be an outcome under runoff rule: Candidate n cannot win in the second round, as any opponent is majority-preferred to Candidate n. Furthermore, it is not possible for a^n to win more than 50% of the votes in the first round. If all candidate choose a^n, then each receives $1/n \leq 0.5$ of the votes. If one candidate chooses $a^i \neq a^n$, and a^n wins, then this implies that a^n would also win against a^i in a two-candidate competition, i.e., $a^n \succ^* a^i$ which contradicts the assumption that a^n is a Condorcet loser.

Let a^i be the the plurality rule winner. If the runoff election does not go to the second round, then a^i must win more than 50% of the votes under plurality. In this case there is no difference in the outcome between the two types of elections. If a^i wins less than 50% under plurality then under runoff a^i will be in the second round competing against a policy a^j. Thus, in the second round of runoff a^j will win if $a^j \succ^* a^i$, i.e., the outcome under runoff, a^j, is majority preferred to the outcome under plurality rule. ■

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